* if ***A*** has a **left-inverse** (***BA=I***) and a **right-inverse** (***AC=I***), the **two inverses** are **equal**
* **an inverse exists only when the rank is as large as possible.**
* **only a square matrix has a two-sided inverse**
* **rectangular matrix cannot have both existence and uniqueness**
* **square matrix cannot have one property without the other.**
  + **existence implies uniqueness and uniqueness implies existence**
* **existence:** **full row rank *r = m***
  + ***Ax=b*** has **at lesat** **one solution** *x* for every *b* if&only if columns span ***Rm***
    - ***x = Cb***  (***Ax = ACb = b)*** there are many ***C***’s,
    - **the number** of solutions :**[1, +infinite)**
  + **right-inverse *C***: ***AT(AAT)-1 (AAT has an inverse, if rank is m)***
    - ***AC=Im* (m by m)**
* **uniqueness: full column rank *r = n***
  + ***Ax=b*** has **at most** **one solution** *x* for every *b* if&only if columns are independent
    - ***x = Bb***  (***x = BAx = Bb)*** there are many ***B***’s
    - **when *r = n* there are no free variables, so the number** of solutions :***0 or 1***
  + **left-inverse *B***: ***(ATA)-1AT (ATA has an inverse, if rank is n)***
    - ***BA=In* (n by n)**
* **the condition** for invertibility is **full rank: *r = m = n***
* **each** of these conditions is a **necessary and sufficient test**
  + 1. the columns span ***Rm***, so ***A****x = b* has **at least** one solution for every *b*
  + 2. the columns are independent, so ***A****x = 0* has **only** the solution *x = 0*
  + 3. the rows of ***A*** span ***Rn***
  + 4. the rows are independent
  + 5. elimination can be completed: ***PA*** *=* ***LDU***, with all *n* pivots
  + **6. the determinant of *A* is not zero**
  + **7. zero is not an eigenvalue of A**
  + *8.* ***ATA*** is positive definite

**Matrices of Rank 1**

* **every matrix of rank 1 has the simple form *A = uvT* = column times row**